

# Double Froggatt–Nielsen Mechanism

Kenji Nishiwaki,<sup>1\*</sup> Yusuke Shimizu,<sup>1,2,3†</sup> and Yoshiyuki Tatsuta<sup>4‡</sup>

<sup>1</sup>*School of Physics, KIAS, Seoul 130-722, Republic of Korea*

<sup>2</sup>*Quantum Universe Center, KIAS, Seoul 130-722, Republic of Korea*

<sup>3</sup>*Graduate School of Science, Hiroshima University, Higashi-Hiroshima, 739-8526, Japan*

<sup>4</sup>*Department of Physics, Waseda University, Tokyo 169-8555, Japan*

## Abstract

We present a doubly parametric extension of the standard Froggatt–Nielsen (FN) mechanism. As is well known, mass matrices of the up- and down-type quark sectors and the charged lepton sector in the standard model can be parametrized well by a parameter  $\lambda$  which is usually taken to be the sine of the Cabibbo angle ( $\lambda = \sin \theta_C \simeq 0.225$ ). However, in the neutrino sector, there is still room to realize the two neutrino mass squared differences  $\Delta m_{\text{sol}}^2$  and  $\Delta m_{\text{atm}}^2$ , two large mixing angles  $\theta_{12}$  and  $\theta_{23}$ , and non-zero  $\theta_{13}$ . Then we consider an extension with an additional parameter  $\rho$  in addition to  $\lambda$ . Taking the relevant FN charges for a power of  $\lambda$  ( $= 0.225$ ) and additional FN charges for a power of  $\rho$ , which we assume to be less than one, we can reproduce the ratio of the two neutrino mass squared differences and three mixing angles. In the normal neutrino mass hierarchy, we show several patterns for taking relevant FN charges and the magnitude of  $\rho$ . We find that if  $\sin \theta_{23}$  is measured more precisely, we can distinguish each pattern. This is testable in the near future, for example in neutrino oscillation experiments. In addition, we predict the Dirac CP-violating phase for each pattern.

---

\*E-mail address: nishiken@kias.re.kr

†E-mail address: yu-shimizu@hiroshima-u.ac.jp

‡E-mail address: y\_tatsuta@akane.waseda.jp

# 1 Introduction

The standard model (SM) is one of the successful models in explaining the results of recent precise experiments. However, there are many free parameters, particularly as Yukawa couplings in the SM. There are some ambiguities in realizing the quark and lepton mass hierarchies and mixing angles. Then, many authors have studied texture analyses or flavor symmetry models in order to elucidate the origin of the flavor structure as a direction beyond the SM. In fact, Weinberg proposed a simple zero texture, within two generations of quarks, where quark masses and a mixing angle are related [1]. Fritzsche extended this to three generations in the so-called “Fritzsch-type mass matrix” [2, 3], which relates quark masses and mixing angles in the quark sector. Furthermore, Fukugita, Tanimoto, and Yanagida extended this argument to the lepton sector [4] and predicted two large neutrino mixing angles and non-zero  $\theta_{13}$  [5, 6], which was the last mixing angle of the lepton sector [7]–[9].

On the other hand, flavor symmetry also plays an important role in understanding the flavor structure. Froggatt and Nielsen proposed the so-called “Froggatt–Nielsen (FN) mechanism” [10], which introduces  $U(1)_{\text{FN}}$  symmetry as flavor symmetry. Taking relevant  $U(1)_{\text{FN}}$  charge assignments to the different generations, the quark mass hierarchy and Cabibbo–Kobayashi–Maskawa (CKM) matrix are naturally reproduced in the quark sector, while the non-Abelian discrete flavor symmetries [11]–[14] can easily derive the large mixing angles in the lepton sector, e.g., tri-bimaximal (TBM) mixing [15, 16], which is a mixing paradigm by describing simple mass textures. After the reactor neutrino experiments reported non-zero  $\theta_{13}$ , it is important to study other flavor paradigms, e.g., tri-bimaximal–Cabibbo (TBC) mixing [17, 18], with the same mindset as TBM, bi-maximal (BM), tri-maximal, and golden ratio neutrino mixing. (For a review, see [11]–[14]). Thus the texture analysis and the flavor symmetry are important in understanding the flavor structures for both quark and lepton sectors and there are many works known as, e.g., “stitching the Yukawa quilt” [19], “ $\mu$ – $\tau$  anarchy” [20, 21], “cascades” [22], “Occam’s razor” [23, 24], and “repressing anarchy” [25, 26].

As is well known, mass matrices of the up- and down-type quarks and the charged leptons in the SM can be parametrized well by a parameter  $\lambda$ , which is usually taken to be the sine of the Cabibbo angle. Taking  $\lambda \simeq 0.225$ , up- and down-type quark and charged lepton mass hierarchies and mixing angles are reproduced. This type of parametrization was originally proposed by Froggatt and Nielsen [10]. On the other hand, however, in the neutrino sector, there is still room to realize the lepton flavor structure, i.e., neutrino mass squared differences  $\Delta m_{\text{sol}}^2$  and  $\Delta m_{\text{atm}}^2$ , two large mixing angles  $\theta_{12}$  and  $\theta_{23}$ , and non-zero  $\theta_{13}$ . Indeed, it is likely that the neutrino mass matrix has a property distinct from those of the up- and down-type quarks and the charged lepton mass matrices. This is due to the fact that the neutrino masses are so tiny in comparison with the other SM fermion masses, and that the lepton mixing angles are relatively larger than the quark mixing angles. In this paper, we present an extension of the FN mechanism in the neutrino mass matrix. In particular, we focus on a doubly parametric extension of the FN (DFN) mechanism.<sup>1</sup> To explain, we show an illustrative example of the doubly parametric extension. This extension is plausible when we use the seesaw mechanism [29]–[34], for instance. If the neutrinos are Majorana particles,

---

<sup>1</sup>In some models of the up- and down-type quark sectors, it is known that the mass matrices are consequently parametrized by two parameters. For example, see Refs. [27, 28].

in the seesaw mechanism, we need both Dirac and Majorana mass terms. Even if the Dirac neutrino mass matrix is parametrized by  $\lambda$  like the other SM fermions, where the Dirac-type masses come from spontaneous symmetry breaking in the SM, the Majorana masses can include free mass parameters in general, and in some models it is plausible that Majorana masses are parametrized by another parameter. Then in the neutrino sector, such a situation corresponds to an extended FN mechanism with a parameter  $\rho$  in addition to  $\lambda$ .<sup>2</sup> Taking the relevant FN charges for the power of  $\lambda$  ( $= 0.225$ ) and additional FN charges for the power of  $\rho$ , which we assume to be less than one, we can reproduce the ratio of two neutrino mass squared differences and three mixing angles. In our numerical calculations, we show several patterns for taking relevant FN charges and the magnitude of  $\rho$ . Note that in our numerical analyses, we consider only the normal neutrino mass hierarchy. We find that if  $\sin \theta_{23}$  is measured more precisely, we can distinguish each pattern. In addition, we predict the Dirac CP-violating phase ( $\delta_{\text{CP}}$ ) for each pattern.

This paper is organized as follows. In Sect. 2, we show the standard FN mechanism and present the DFN mechanism. In Sect. 3, we show the results of our numerical analyses in several patterns. Section 4 is devoted to discussions and summary. In Appendix A, we show the explicit form of the neutrino mass matrix for each pattern.

## 2 Doubly parametric extension of the FN mechanism

It is known that the mass matrices of the up- and down-type quark sectors and the charged lepton sector in the SM can be parametrized well by a parameter  $\lambda$  and six charges  $\{a_i, b_j\}$  ( $i, j = 1, 2, 3$ ), i.e.,

$$m_{ij} = \lambda^{a_i+b_j}, \quad (1)$$

with up to  $\mathcal{O}(1)$  complex coefficients in front of each element.<sup>3</sup> In particular, it is reasonable to choose a value of the parameter  $\lambda$  such that the observed masses and mixing angles of the up- and down-type quark and the charged lepton sectors can be realized. Indeed, such a value is given as  $\lambda = \sin \theta_C \simeq 0.225$ , where  $\theta_C$  is the Cabibbo angle. This type of parametrization was originally proposed by Froggatt and Nielsen, the so-called “FN parametrization” [10].

In this paper, we consider an extension with an additional parameter  $\rho$  and six additional charges  $\{c_i, d_j\}$  ( $i, j = 1, 2, 3$ ), i.e.,

$$m_{ij} = \lambda^{a_i+b_j} \rho^{c_i+d_j}, \quad (2)$$

also with up to  $\mathcal{O}(1)$  complex coefficients in front of each element. In particular, this parametrization is valid in a neutrino mass matrix as well as the other fermion mass matrices.

---

<sup>2</sup>Note that we show another extension of the FN mechanism, the Gaussian FN mechanism on magnetized orbifolds in Ref. [35].

<sup>3</sup>We note that this form can be derived from extra dimensions. When we assume that the SM fermions propagate in the bulk of an interval and have Yukawa couplings on the brane at  $y = L$ , the mass matrix is symbolically written down as  $m_{ij} \propto e^{L(M_{L_i} - M_{R_j})} \sim \lambda^{a_i+b_j}$ , where  $L$  is the length of the interval.  $M_{L_i/R_i}$  are bulk masses for  $i$ th-generation doublet/singlet, and we put the Dirichlet boundary condition for right/left-handed mode of the doublets/singlets at the two end points at  $y = 0, L$  to realize left-hand doublet modes and right-hand singlet modes, respectively. We adopt the notation in Ref. [36]. If the particle profiles are also localized among other directions of extra dimensions, we might address the DFN structure.

It should be noted that there are some possibilities for realizing the DFN parametrization. For example, the DFN parametrization would be considered as effective theories of multi-scale extra dimensions, and would be obtained by an additional  $U(1)$  flavor symmetry and so on. To construct concrete models including the DFN parametrization is beyond the scope of this paper. In Refs. [27, 28], for the up- and down-type quark sectors, the phenomenological prospects of the doubly parametric extension have already been studied. In this paper, we focus only on phenomenological properties of the doubly parametric extension in the lepton sector, in particular the neutrino mass matrix. Here, we assume that the charged lepton mass matrix takes a diagonal form.

Finally, it is important to comment on concrete values of the two parameters  $\lambda$  and  $\rho$ . Note that without loss of generality we can choose the value of the original parameter  $\lambda$  such that  $\lambda = 0.225$ . Even if the parameter is chosen to be a distinct value, we can move to the case of  $\lambda = 0.225$  by redefining the additional FN charges  $\{c_i, d_j\}$  and the value of the additional parameter  $\rho$ . Hence, in the following, we take  $\lambda = 0.225$  and  $\rho$  as an arbitrary value which we assume to be less than one. We show that the DFN textures can reproduce the ratio of the two neutrino mass squared differences and three mixing angles. We also show the results of our numerical analyses in the next section. Here, we do not identify the origin of the additional parameter  $\rho$ , where one possibility is (right-handed) Majorana neutrino mass parameters in a seesaw model.

### 3 Numerical analyses

In this section, we focus on mass matrices in the neutrino sector, and also analyze numerical aspects of the extended FN parametrization. In our numerical calculations, we assume the normal neutrino mass hierarchy. We use the results of the global analysis of neutrino oscillation experiments [37]. The  $3\sigma$  ranges of the experimental data for the normal neutrino mass hierarchy are given as

$$\begin{aligned} 0.270 \leq \sin^2 \theta_{12} \leq 0.344, \quad 0.382 \leq \sin^2 \theta_{23} \leq 0.643, \quad 0.0186 \leq \sin^2 \theta_{13} \leq 0.0250, \\ 7.02 \leq \frac{\Delta m_{\text{sol}}^2}{10^{-5} \text{ eV}^2} \leq 8.09, \quad 2.317 \leq \frac{\Delta m_{\text{atm}}^2}{10^{-3} \text{ eV}^2} \leq 2.607, \end{aligned} \quad (3)$$

where  $\theta_{ij}$  are lepton mixing angles in the Pontecorvo–Maki–Nakagawa–Sakata (PMNS) matrix, while  $\Delta m_{\text{sol}}^2$  and  $\Delta m_{\text{atm}}^2$  are the solar and atmospheric neutrino mass squared differences, respectively. Note that the DFN parametrization can be valid in both Dirac and Majorana mass matrices. For simplicity, we assume that FN charges  $a_i(c_i)$  are equivalent to the other charges  $b_j(d_j)$ , respectively. Now, the mass matrix in Eq. (2) becomes symmetric. We also assume that the mass matrix of the charged leptons is diagonal.<sup>4</sup>

In the following, we consider six patterns<sup>5</sup> with/without additional FN charges as sample

---

<sup>4</sup>It is clear that we can consider large contributions to lepton mixing angles, e.g., Ref. [38]. However, such large contributions are not typical setups in the framework of the FN parametrization. In many typical cases, the contributions from the charged lepton mass matrices are considered to be small enough to be negligible.

<sup>5</sup>We analyzed various textures and picked six patterns to be mentioned where the result of the recent neutrino oscillation experiments [37] tends to be explained in part of parameter space.

patterns for numerical calculation:<sup>6</sup>

- Pattern 1 :  $a_i = \{1, 0, 0\}$ ,  $\forall c_i$ , and  $\rho = 1.0$ ;
- Pattern 2 :  $a_i = \{1, 0, 0\}$ ,  $c_i = \{0, \frac{3}{2}, \frac{5}{2}\}$ , and  $\rho = 0.8$ ;
- Pattern 3 :  $a_i = \{\frac{3}{2}, \frac{1}{2}, 0\}$ ,  $c_i = \{0, \frac{1}{2}, \frac{3}{2}\}$ , and  $\rho = 0.4$ ;
- Pattern 4 :  $a_i = \{\frac{3}{2}, \frac{1}{2}, 0\}$ ,  $c_i = \{0, \frac{1}{2}, \frac{3}{2}\}$ , and  $\rho = 0.5$ ;
- Pattern 5 :  $a_i = \{\frac{3}{2}, \frac{1}{2}, 0\}$ ,  $c_i = \{0, \frac{1}{2}, \frac{3}{2}\}$ , and  $\rho = 0.6$ ;
- Pattern 6 :  $a_i = \{\frac{1}{2}, \frac{1}{2}, 0\}$ ,  $c_i = \{1, 0, \frac{1}{2}\}$ , and  $\rho = 0.3$ .

Note that with  $\rho = 1.0$ , the first pattern of charge configurations gives the standard FN parametrization, and this charge configuration gives a  $\mu$ - $\tau$  symmetric mass matrix which derives the almost BM mixing. In our calculations, the three neutrino masses are adjusted by the ratio of the two neutrino mass squared differences, because an overall mass scale is completely free for our parametrization. Here we take  $\mathcal{O}(1)$  coefficients as 10% deviations from unity and complex phases are taken from  $-\pi$  to  $\pi$ . Then, we can predict the Dirac CP-violating phase  $\delta_{\text{CP}}$  in our numerical calculations, where the non-zero  $\delta_{\text{CP}}$  originates from the complex phases of the mass matrix elements. In each pattern, we scan  $10^6$  configurations of the coefficients of the nine elements of the mass matrix.

First, we show the scatter plots in Pattern 1. The gray regions suggest realized values of mixing angles  $\sin \theta_{12}$ ,  $\sin \theta_{23}$ ,  $\sin \theta_{13}$ , and Dirac CP-violating phase  $\delta_{\text{CP}}$  in Fig. 1. The insides of the red dotted lines show the  $3\sigma$  allowed regions of each lepton mixing angle, while the orange points correspond to the case that all three lepton mixing angles are within the  $3\sigma$  ranges simultaneously in Eq. (3). We find that  $\delta_{\text{CP}}$  is predicted as  $|\delta_{\text{CP}}| \lesssim 1$  and  $2.2 \lesssim |\delta_{\text{CP}}| \lesssim \pi$ .

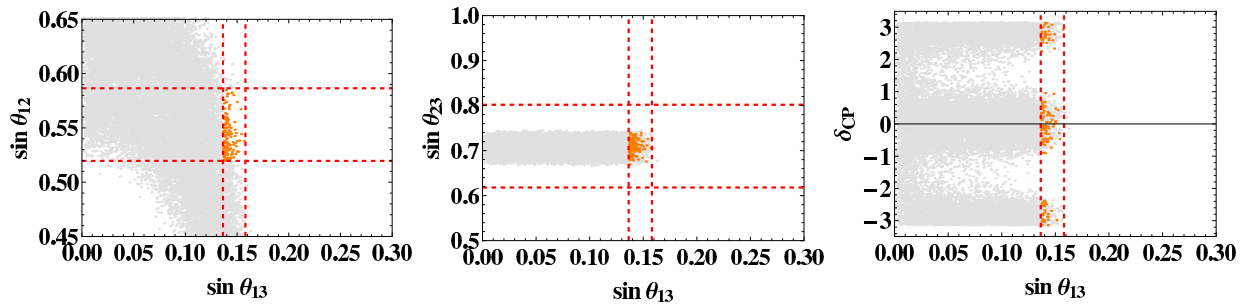


Figure 1: Scatter plots in Pattern 1:  $a_i = \{1, 0, 0\}$ ,  $\forall c_i$ . We set  $\rho = 1.0$ . The ratio of the two neutrino mass squared differences is required within the range where  $\Delta m_{\text{sol}}^2$  and  $\Delta m_{\text{atm}}^2$  are inside the  $3\sigma$  ranges shown in Eq. (3). The insides of the red dotted lines show the  $3\sigma$  allowed regions of the lepton mixing angles. The orange points correspond to the case that all three lepton mixing angles are within the  $3\sigma$  ranges simultaneously in Eq. (3).

<sup>6</sup>We show the explicit form of the mass matrix of neutrinos for each pattern in Appendix A.

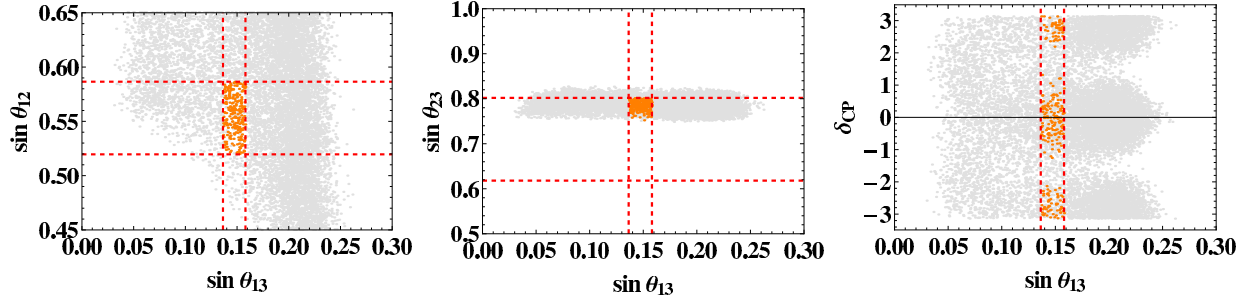


Figure 2: Scatter plots in Pattern 2:  $a_i = \{1, 0, 0\}$ ,  $c_i = \{0, \frac{3}{2}, \frac{5}{2}\}$ . We set  $\rho = 0.8$ . The color convention is as in Fig. 1.

On the other hand, in Pattern 2, the standard FN charges are the same as those of Pattern 1 and we set  $\rho = 0.8$  so that the mass matrix becomes almost  $\mu$ - $\tau$  symmetric (though not exactly). Figure 2 shows that  $\sin \theta_{23}$  is around the upper boundary of the  $3\sigma$  range, while the other mixing angles are completely filled within the  $3\sigma$  range. Comparing Figures 1 and 2, it is easily seen that the realized values of  $\sin \theta_{13}$  in Pattern 2 are relatively larger than those in Pattern 1. This implies that  $\sin \theta_{13}$  is improved by the DFN parametrizations, even if the coefficients are not so scattered in large parameter regions. Also, we can distinguish Patterns 1 and 2 between the standard FN and DFN by more precise measurement of  $\sin \theta_{23}$ . The extension with an additional parameter leads to modestly different properties of  $\sin \theta_{13}$  and  $\sin \theta_{23}$  from those of a  $\mu$ - $\tau$  symmetric neutrino mass matrix.

Next, we show other patterns from non- $\mu$ - $\tau$  symmetric neutrino mass matrices. In Patterns 3, 4, and 5, the charge configurations of  $a_i$  and  $c_i$  are  $a_i = \{\frac{3}{2}, \frac{1}{2}, 0\}$ ,  $c_i = \{0, \frac{1}{2}, \frac{3}{2}\}$ , while the magnitudes of  $\rho$  are different:  $\rho = 0.4, 0.5, 0.6$ , respectively. If we set  $\rho = 1.0$  which is the standard FN parametrization, we cannot find the correct ratio of the two neutrino mass squared differences and three mixing angles. In Figs. 3, 4, and 5, the allowed regions of  $\sin \theta_{12}$ ,  $\sin \theta_{13}$ , and  $\delta_{CP}$  are almost the same, while  $\sin \theta_{23}$  is completely different. In Figs. 3, 4 and 5, it is easily found that different values of  $\rho$  lead to different values of  $\sin \theta_{23}$ . This is a remarkable property in the DFN parametrizations. Figure 3 shows that  $\sin \theta_{23}$  is scattered around the upper boundary of the  $3\sigma$  range in Pattern 3. In Pattern 4, the allowed region of  $\sin \theta_{23}$  is  $0.63 \lesssim \sin \theta_{23} \lesssim 0.72$ , as shown in Fig. 4. In Pattern 5, Fig. 5 shows that  $\sin \theta_{23}$  is around the lower boundary of the  $3\sigma$  range. When we set  $\rho = 0.3$  or  $0.7$ , the obtained values of  $\sin \theta_{23}$  are beyond the  $3\sigma$  experimental upper and lower bounds, respectively. The three patterns are tested by measuring the value of  $\sin \theta_{23}$  more precisely.



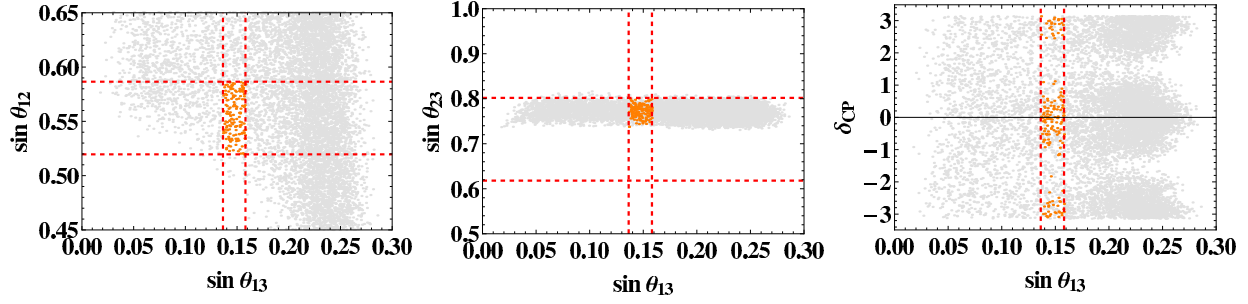


Figure 3: Scatter plots in Pattern 3:  $a_i = \{\frac{3}{2}, \frac{1}{2}, 0\}$ ,  $c_i = \{0, \frac{1}{2}, \frac{3}{2}\}$ . We set  $\rho = 0.4$ . The color convention is as in Fig. 1.

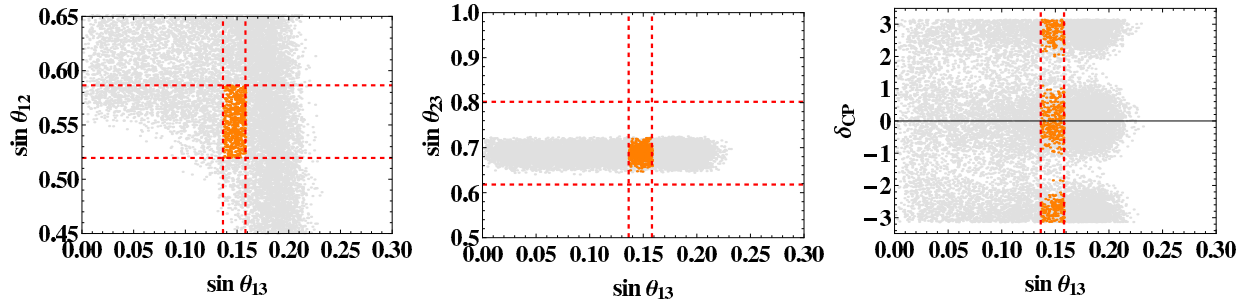


Figure 4: Scatter plots in Pattern 4:  $a_i = \{\frac{3}{2}, \frac{1}{2}, 0\}$ ,  $c_i = \{0, \frac{1}{2}, \frac{3}{2}\}$ . We set  $\rho = 0.5$ . The color convention is as in Fig. 1.

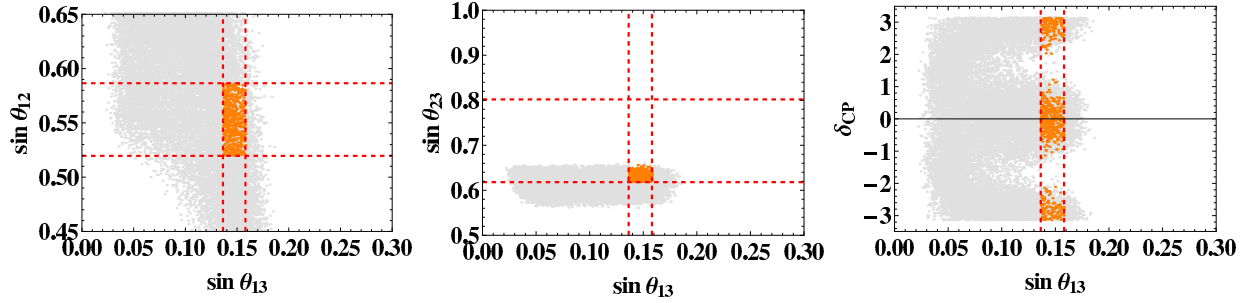


Figure 5: Scatter plots in Pattern 5:  $a_i = \{\frac{3}{2}, \frac{1}{2}, 0\}$ ,  $c_i = \{0, \frac{1}{2}, \frac{3}{2}\}$ . We set  $\rho = 0.6$ . The color convention is as in Fig. 1.

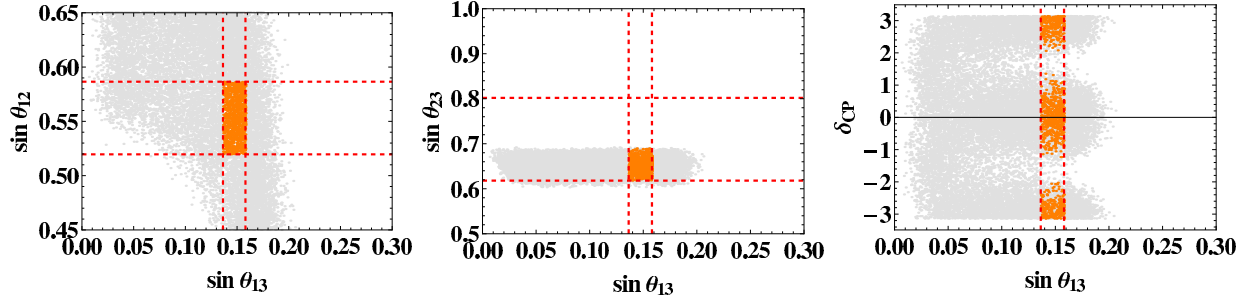


Figure 6: Scatter plots in Pattern 6:  $a_i = \{\frac{1}{2}, \frac{1}{2}, 0\}$ ,  $c_i = \{1, 0, \frac{1}{2}\}$ . We set  $\rho = 0.3$ . The color convention is as in Fig. 1.

Finally, we show the last pattern. In Pattern 6 we set  $\rho = 0.3$ , which means that this pattern seems to be almost the standard FN parametrization because of  $\lambda = 0.225$ .<sup>7</sup> In Fig. 6,  $\sin \theta_{12}$  and  $\sin \theta_{13}$  are filled within the  $3\sigma$  range, while  $\sin \theta_{23}$  is distributed around the lower boundary of the  $3\sigma$  range. Note that the neutrino mass matrix in Pattern 6 is considered to be similar to that in Pattern 1, because the value of the additional parameter is small and approximately equal to  $\lambda$ , i.e.,  $\rho = 0.3$ . However, the values obtained for the mixing angles are distinct from each other. Indeed, the DFN extension can make values of  $\sin \theta_{13}$  larger and values of  $\sin \theta_{23}$  relatively smaller. We recognize that these properties are distinctive from those of Pattern 1 (the original FN). In addition, we find that  $\delta_{CP}$  is predicted as  $|\delta_{CP}| \lesssim 1$  and  $2.0 \lesssim |\delta_{CP}| \lesssim \pi$  for Pattern 6.

We comment on the testabilities of the configurations of the (double) FN charges and parameters. First, the values obtained for  $\sin \theta_{23}$  are almost the same in Patterns 1 and 4. The frequency of consistent values of  $\sin \theta_{13}$  is certainly improved in Pattern 4. However, the predicted values are dependent on the coefficients in front of each element. Hence, it is difficult to distinguish Patterns 1 and 4 by neutrino experimental data. Indeed, such situations happen between the other patterns, e.g., between Patterns 2 and 3 and between Patterns 5 and 6. These coincident properties of predicted mixing angles can also be seen in the standard FN parametrizations. Therefore, all of the patterns cannot always be tested by more precise determination of the three mixing angles. This is more conspicuous when  $\mathcal{O}(1)$  coefficients are randomly scattered in a wider range, e.g.,  $c_{ij} \in [0.8, 1.2]$ . The testability of between different configurations of (double) FN charges and parameters is strongly dependent on concrete model building.

## 4 Discussions and summary

In the SM, there are many free parameters especially as Yukawa couplings, so that there are some ambiguities in realizing the quark and lepton mass hierarchies and mixing angles. It is therefore important to study texture analysis or a flavor symmetry model in order to elucidate the origin of the flavor structure as beyond the SM. As is well known, mass matrices of the up- and down-type quarks and the charged leptons in the SM can be parametrized well by the

<sup>7</sup>If we set  $\rho = 1.0$ , we cannot find the correct neutrino masses and mixing angles in the charge configurations of  $a_i$  and  $c_i$  for Pattern 6. Then the DFN parametrization can be well parametrized in the neutrino sector.



parameter  $\lambda$  which is usually taken to be the sine of the Cabibbo angle ( $\lambda = \sin \theta_C \simeq 0.225$ ). In this parametrization, the mass hierarchies and mixing angles of the quarks and charged leptons are reproduced. However, in the neutrino sector, there is still room to realize the neutrino mass squared differences  $\Delta m_{\text{sol}}^2$  and  $\Delta m_{\text{atm}}^2$ , two large mixing angles  $\theta_{12}$  and  $\theta_{23}$ , and non-zero  $\theta_{13}$ . Actually, if the neutrinos are Majorana particles, in the seesaw mechanism, we need both the Dirac and Majorana mass terms. Even if the Dirac neutrino mass matrix is parametrized by  $\lambda$  like the other SM fermions, the Majorana masses include free mass parameters in general, and it is plausible that Majorana masses are parametrized by another parameter. Thus, in this paper we have presented a doubly parametric extension of the FN mechanism with the parameter  $\rho$  in addition to  $\lambda$ . Taking the relevant FN charges for the power of  $\lambda$  ( $= 0.225$ ) and additional FN charges for the power of  $\rho$ , which we assume to be less than one, we can reproduce the ratio of the neutrino mass squared differences and lepton mixing angles. Here we assume that the charged lepton mass matrix is diagonal.

In our calculations, the neutrino masses, assuming the normal neutrino mass hierarchy, are adjusted by the ratio of the neutrino mass squared differences because the overall mass scale is completely free for our parametrization. Here we take  $\mathcal{O}(1)$  coefficients as 10% deviations from one and the complex phases are taken from  $-\pi$  to  $\pi$ . Note that if we take the magnitude of  $\mathcal{O}(1)$  coefficients as 20% deviations from one, the allowed region of  $\delta_{\text{CP}}$  is  $-\pi \lesssim \delta_{\text{CP}} \lesssim \pi$ , while if we take the magnitude of  $\mathcal{O}(1)$  coefficients as 5% deviations from one,  $\delta_{\text{CP}}$  is more predictive. In this paper, we considered six patterns with/without additional FN charges as sample patterns for numerical calculations. First, we showed the standard FN and DFN parametrizations which are almost  $\mu$ - $\tau$  symmetric mass matrices in Patterns 1 and 2, respectively. We found that  $\delta_{\text{CP}}$  is predicted as  $|\delta_{\text{CP}}| \lesssim 1$  and  $2.2 \lesssim |\delta_{\text{CP}}| \lesssim \pi$ . In Pattern 2,  $\sin \theta_{23}$  is around the upper boundary of the  $3\sigma$  range, while the other mixing angles are completely filled within the  $3\sigma$  range.

Next, we showed other patterns where the neutrino mass matrices are not  $\mu$ - $\tau$  symmetric. In Patterns 3, 4, and 5, the charge configurations of  $a_i$  and  $c_i$  are  $a_i = \{\frac{3}{2}, \frac{1}{2}, 0\}$ ,  $c_i = \{0, \frac{1}{2}, \frac{3}{2}\}$ , while the magnitudes of  $\rho$  are different:  $\rho = 0.4, 0.5, 0.6$ , respectively. If we set  $\rho = 1.0$ , which corresponds to the standard FN parametrization, we cannot find the correct ratio of the two neutrino mass squared differences and three mixing angles, so the DFN pattern parametrizes the neutrino sector well. Finally, we find a sizable deviation in Pattern 6, where the magnitude of  $\rho$  ( $= 0.3$ ) is a little away from the FN value  $\lambda$  ( $\simeq 0.225$ ).

We had seen several examples in the mass matrix form formulated under the concept of DFN. We recognized that patterns of the mixing angles and the Dirac CP phase can deviate from the predicted ones in the FN texture. The deviations look distinctive when fluctuations in the elements of the mass matrix are within 10% of unity. As pointed out in the previous section, the explicit differences between the standard FN and DFN parametrizations tend to appear particularly in values of  $\sin \theta_{23}$ . Hence, measuring  $\sin \theta_{23}$  precisely is achievable in the near future, for example in neutrino oscillation experiments, and such improved measurements can determine how well the DFN texture works.

## Acknowledgements

YS is supported in part by National Research Foundation of Korea (NRF) Research Grant NRF-2015R1A2A1A05001869. This work is supported in part by Grants-in-Aid for Scientific Research [No. 16J05332 (YS) and No. 16J04612 (YT)] from the Ministry of Education,

## A Neutrino mass matrix for each pattern

We show the neutrino mass matrix  $m_\nu^{(i)}$  for each Pattern  $i$ , which we discuss in Sect. 3;

$$\begin{aligned} m_\nu^{(1)} &= \begin{pmatrix} \lambda^2 & \lambda & \lambda \\ \lambda & 1 & 1 \\ \lambda & 1 & 1 \end{pmatrix}, \quad m_\nu^{(2)} = \begin{pmatrix} \lambda^2 & \lambda\rho^{\frac{3}{2}} & \lambda\rho^{\frac{5}{2}} \\ \lambda\rho^{\frac{3}{2}} & \rho^3 & \rho^4 \\ \lambda\rho^{\frac{5}{2}} & \rho^4 & \rho^5 \end{pmatrix}, \quad m_\nu^{(3,4,5)} = \begin{pmatrix} \lambda^3 & \lambda^2\rho^{\frac{1}{2}} & \lambda^{\frac{3}{2}}\rho^{\frac{3}{2}} \\ \lambda^2\rho^{\frac{1}{2}} & \lambda\rho & \lambda^{\frac{1}{2}}\rho^2 \\ \lambda^{\frac{3}{2}}\rho^{\frac{3}{2}} & \lambda^{\frac{1}{2}}\rho^2 & \rho^3 \end{pmatrix}, \\ m_\nu^{(6)} &= \begin{pmatrix} \lambda\rho^2 & \lambda\rho & \lambda^{\frac{1}{2}}\rho^{\frac{3}{2}} \\ \lambda\rho & \lambda & \lambda^{\frac{1}{2}}\rho^{\frac{1}{2}} \\ \lambda^{\frac{1}{2}}\rho^{\frac{3}{2}} & \lambda^{\frac{1}{2}}\rho^{\frac{1}{2}} & \rho^{\frac{3}{2}} \end{pmatrix}, \end{aligned} \tag{4}$$

where the overall mass parameter is omitted for each mass matrix.

## References

- [1] S. Weinberg, Trans. New York Acad. Sci. **38** (1977) 185.
- [2] H. Fritzsch, Phys. Lett. B **73** (1978) 317.
- [3] H. Fritzsch, Nucl. Phys. B **155** (1979) 189.
- [4] M. Fukugita, M. Tanimoto and T. Yanagida, Prog. Theor. Phys. **89** (1993) 263.
- [5] M. Fukugita, M. Tanimoto and T. Yanagida, Phys. Lett. B **562** (2003) 273 [hep-ph/0303177].
- [6] M. Fukugita, Y. Shimizu, M. Tanimoto and T. T. Yanagida, Phys. Lett. B **716** (2012) 294 [arXiv:1204.2389 [hep-ph]].
- [7] F. P. An *et al.* [DAYA-BAY Collaboration], Phys. Rev. Lett. **108** (2012) 171803 [arXiv:1203.1669 [hep-ex]].
- [8] J. K. Ahn *et al.* [RENO Collaboration], Phys. Rev. Lett. **108** (2012) 191802 [arXiv:1204.0626 [hep-ex]].
- [9] Y. Abe *et al.* [Double Chooz Collaboration], Phys. Lett. B **735** (2014) 51 [arXiv:1401.5981 [hep-ex]].
- [10] C. D. Froggatt and H. B. Nielsen, Nucl. Phys. B **147** (1979) 277.
- [11] H. Ishimori, T. Kobayashi, H. Ohki, Y. Shimizu, H. Okada and M. Tanimoto, Prog. Theor. Phys. Suppl. **183** (2010) 1 [arXiv:1003.3552 [hep-th]].
- [12] H. Ishimori, T. Kobayashi, H. Ohki, H. Okada, Y. Shimizu and M. Tanimoto, Lect. Notes Phys. **858** (2012) 1, Springer.

- [13] H. Ishimori, T. Kobayashi, Y. Shimizu, H. Ohki, H. Okada and M. Tanimoto, Fortsch. Phys. **61** (2013) 441.
- [14] S. F. King, A. Merle, S. Morisi, Y. Shimizu and M. Tanimoto, New J. Phys. **16** (2014) 045018 [arXiv:1402.4271 [hep-ph]].
- [15] P. F. Harrison, D. H. Perkins and W. G. Scott, Phys. Lett. B **530** (2002) 167 [hep-ph/0202074].
- [16] P. F. Harrison and W. G. Scott, Phys. Lett. B **535** (2002) 163 [hep-ph/0203209].
- [17] S. F. King, Phys. Lett. B **718** (2012) 136 [arXiv:1205.0506 [hep-ph]].
- [18] Y. Shimizu, R. Takahashi and M. Tanimoto, PTEP **2013** (2013) 6, 063B02 [arXiv:1212.5913 [hep-ph]].
- [19] P. Ramond, R. G. Roberts and G. G. Ross, Nucl. Phys. B **406** (1993) 19 [hep-ph/9303320].
- [20] T. Fukuyama and H. Nishiura, hep-ph/9702253.
- [21] W. Buchmuller and T. Yanagida, Phys. Lett. B **445** (1999) 399 [hep-ph/9810308].
- [22] N. Haba, R. Takahashi, M. Tanimoto and K. Yoshioka, Phys. Rev. D **78** (2008) 113002 [arXiv:0804.4055 [hep-ph]].
- [23] K. Harigaya, M. Ibe and T. T. Yanagida, Phys. Rev. D **86** (2012) 013002 [arXiv:1205.2198 [hep-ph]].
- [24] M. Tanimoto and T. T. Yanagida, arXiv:1601.04459 [hep-ph].
- [25] G. Altarelli, F. Feruglio, I. Masina and L. Merlo, JHEP **1211** (2012) 139 [arXiv:1207.0587 [hep-ph]].
- [26] J. Bergstrom, D. Meloni and L. Merlo, Phys. Rev. D **89** (2014) 9, 093021 [arXiv:1403.4528 [hep-ph]].
- [27] M. Leurer, Y. Nir and N. Seiberg, Nucl. Phys. B **398**, 319 (1993) [hep-ph/9212278].
- [28] M. Leurer, Y. Nir and N. Seiberg, Nucl. Phys. B **420**, 468 (1994) [hep-ph/9310320].
- [29] P. Minkowski, Phys. Lett. B **67** (1977) 421.
- [30] T. Yanagida, in Proceedings of the Workshop on Unified Theories and Baryon Number in the Universe, eds. O. Sawada and A. Sugamoto (KEK report 79-18, 1979).
- [31] M. Gell-Mann, P. Ramond and R. Slansky, in Supergravity, eds. P. van Nieuwenhuizen and D.Z. Freedman (North Holland, Amsterdam, 1979).
- [32] R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44** (1980) 912.

- [33] J. Schechter and J. W. F. Valle, Phys. Rev. D **22** (1980) 2227.
- [34] J. Schechter and J. W. F. Valle, Phys. Rev. D **25** (1982) 774.
- [35] H. Abe, T. Kobayashi, K. Sumita and Y. Tatsuta, Phys. Rev. D **90** (2014) 10, 105006 [arXiv:1405.5012 [hep-ph]].
- [36] Y. Fujimoto, K. Hasegawa, T. Nagasawa, K. Nishiwaki, M. Sakamoto and K. Tatsumi, arXiv:1601.05265 [hep-ph].
- [37] M. C. Gonzalez-Garcia, M. Maltoni and T. Schwetz, arXiv:1512.06856 [hep-ph].
- [38] S. Antusch and S. F. King, Phys. Lett. B **631**, 42 (2005) [hep-ph/0508044].